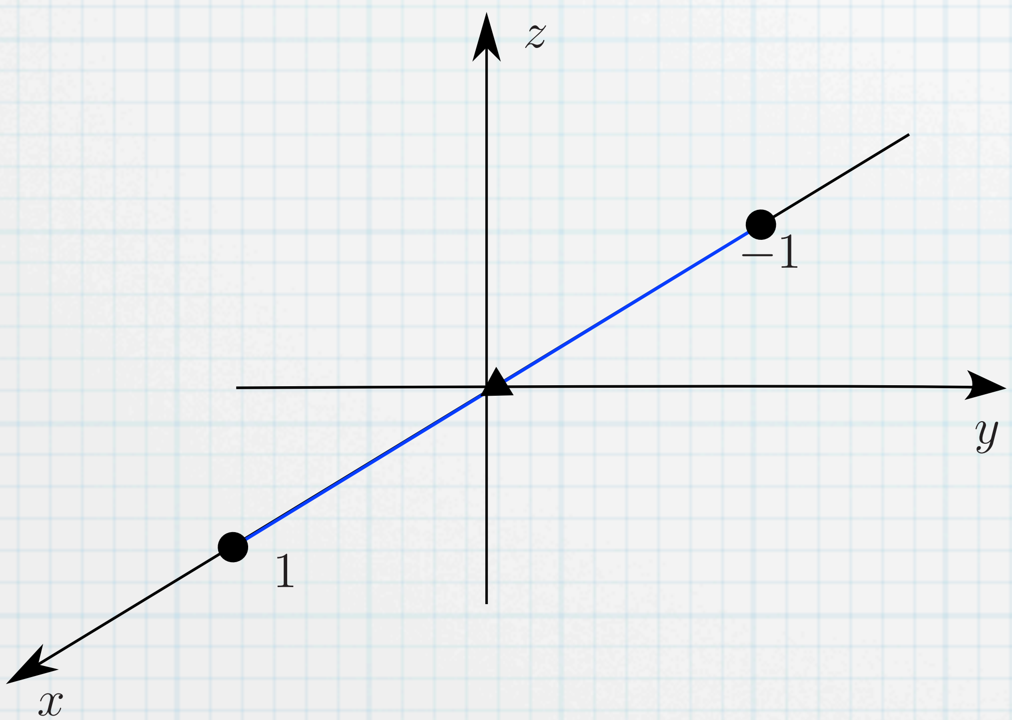
Practica 10

Longitud de arco, Integrales de Trayectoria

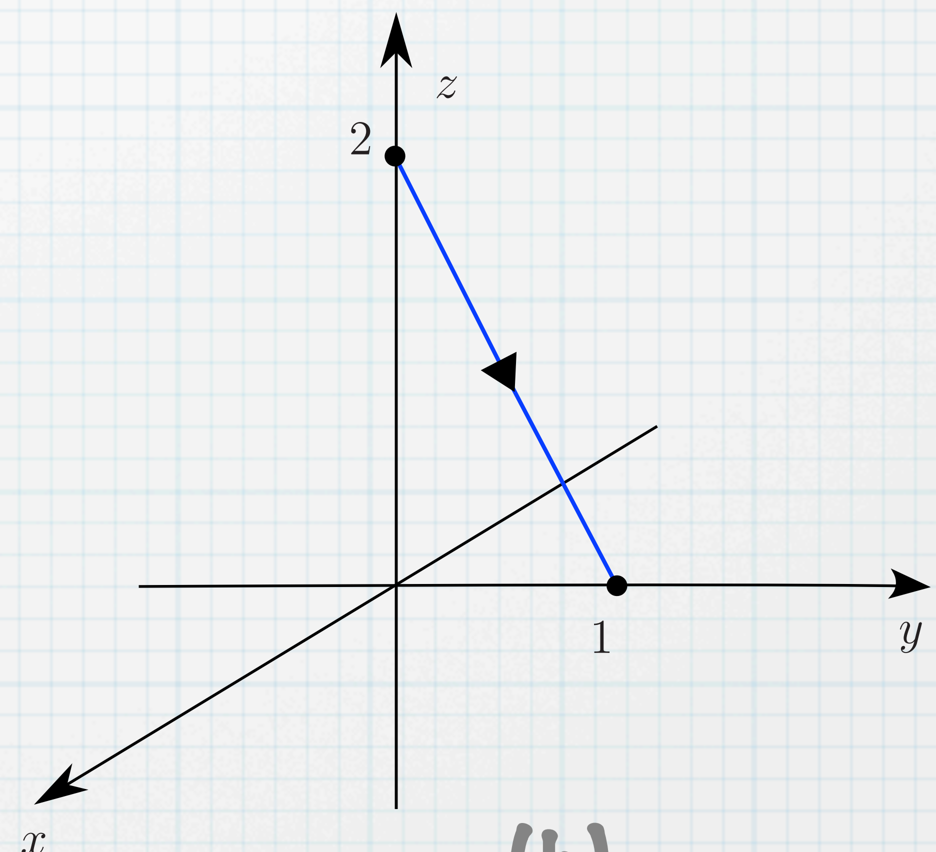
Problema 1.

Trayectorias.

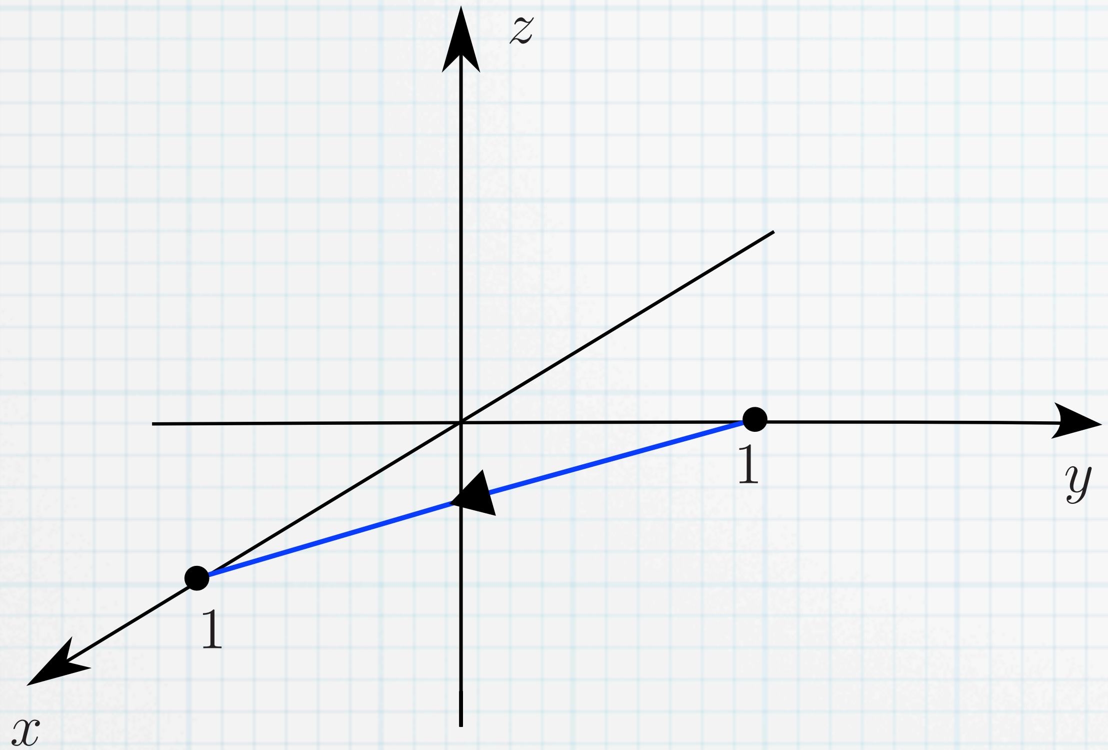
Asocie a cada una de las figuras que se muestran a continuación una ecuación vectorial (1-8)



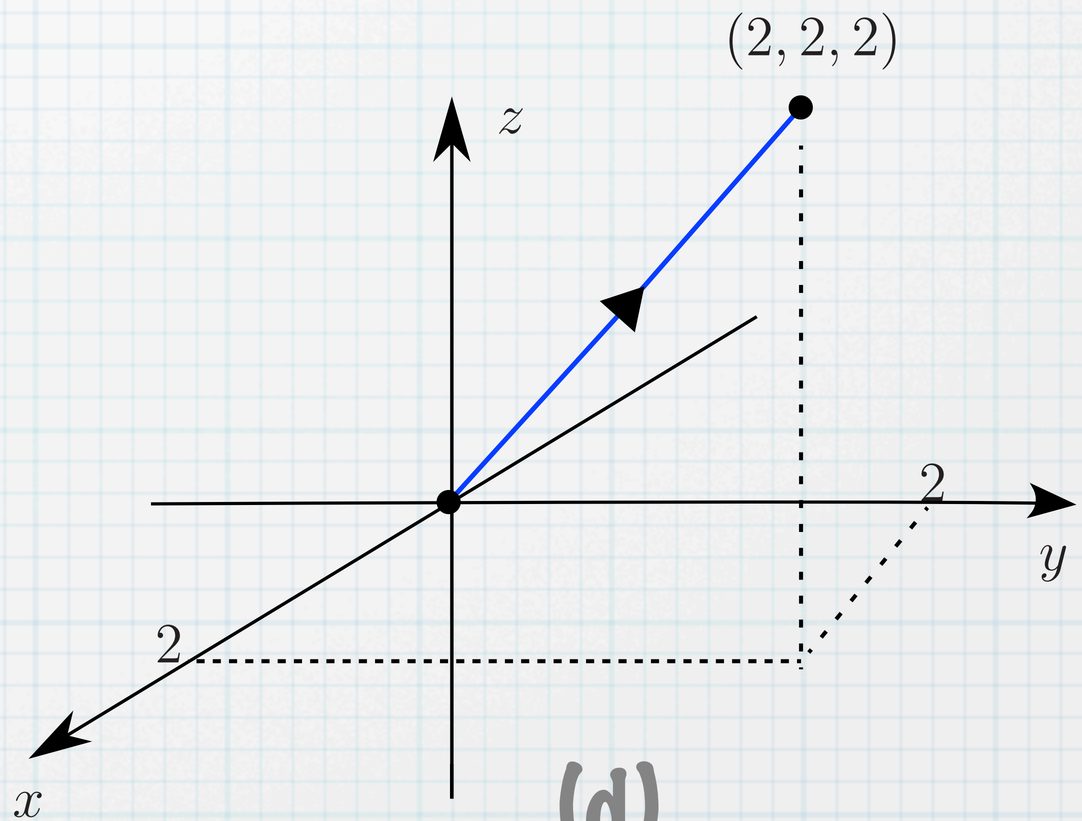
(a)



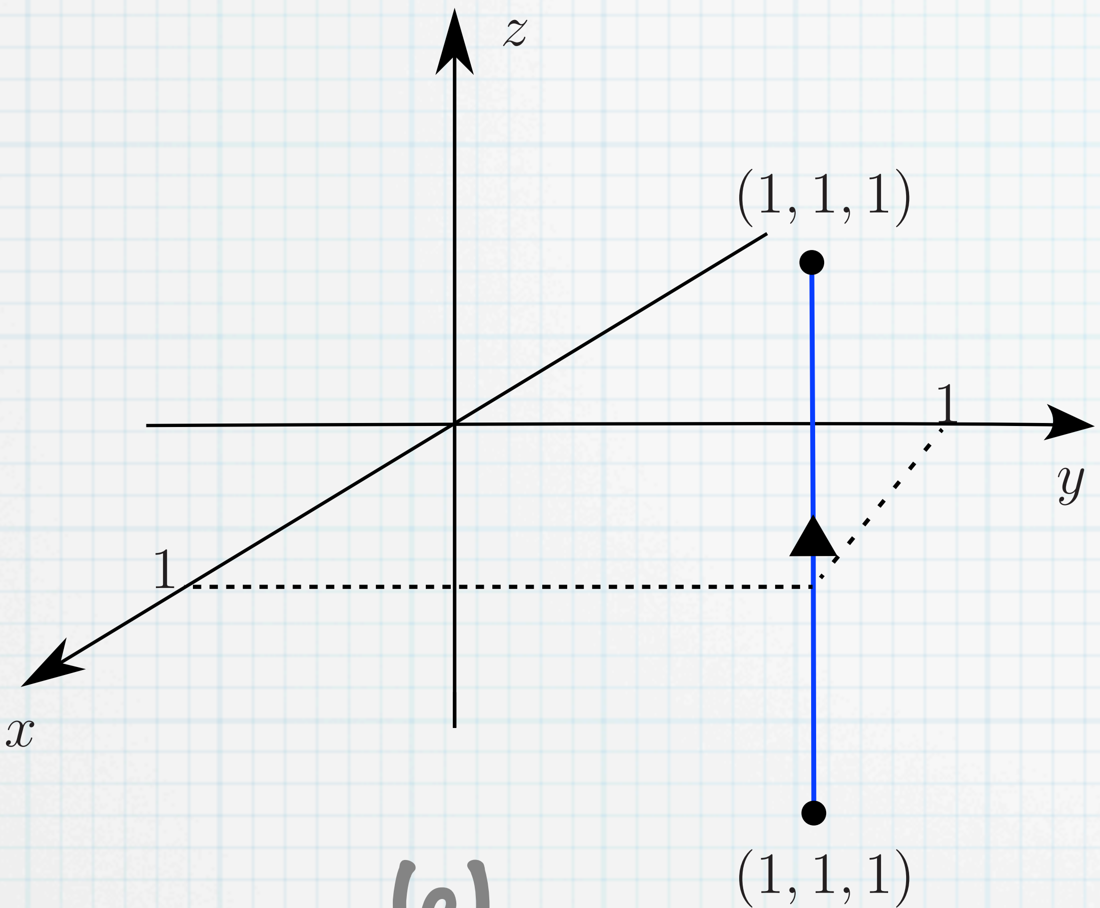
(b)



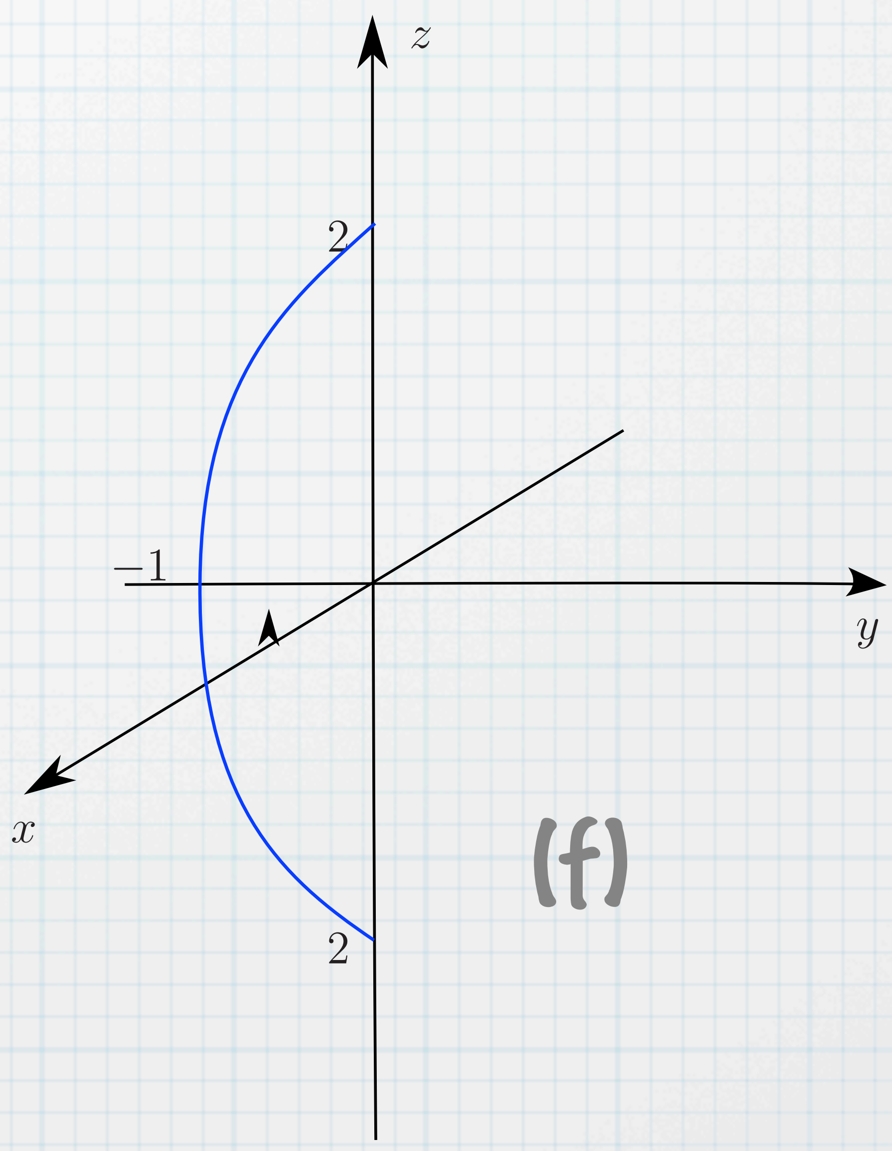
(c)



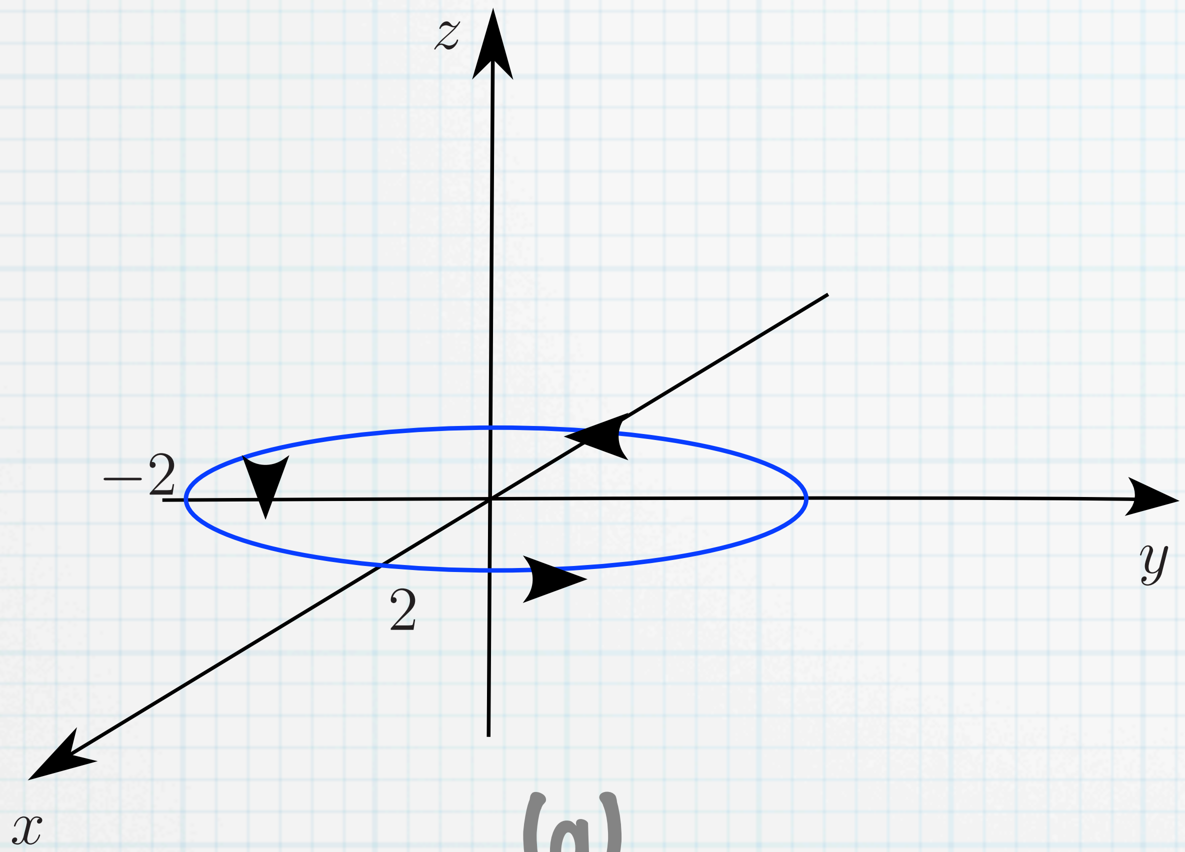
(d)



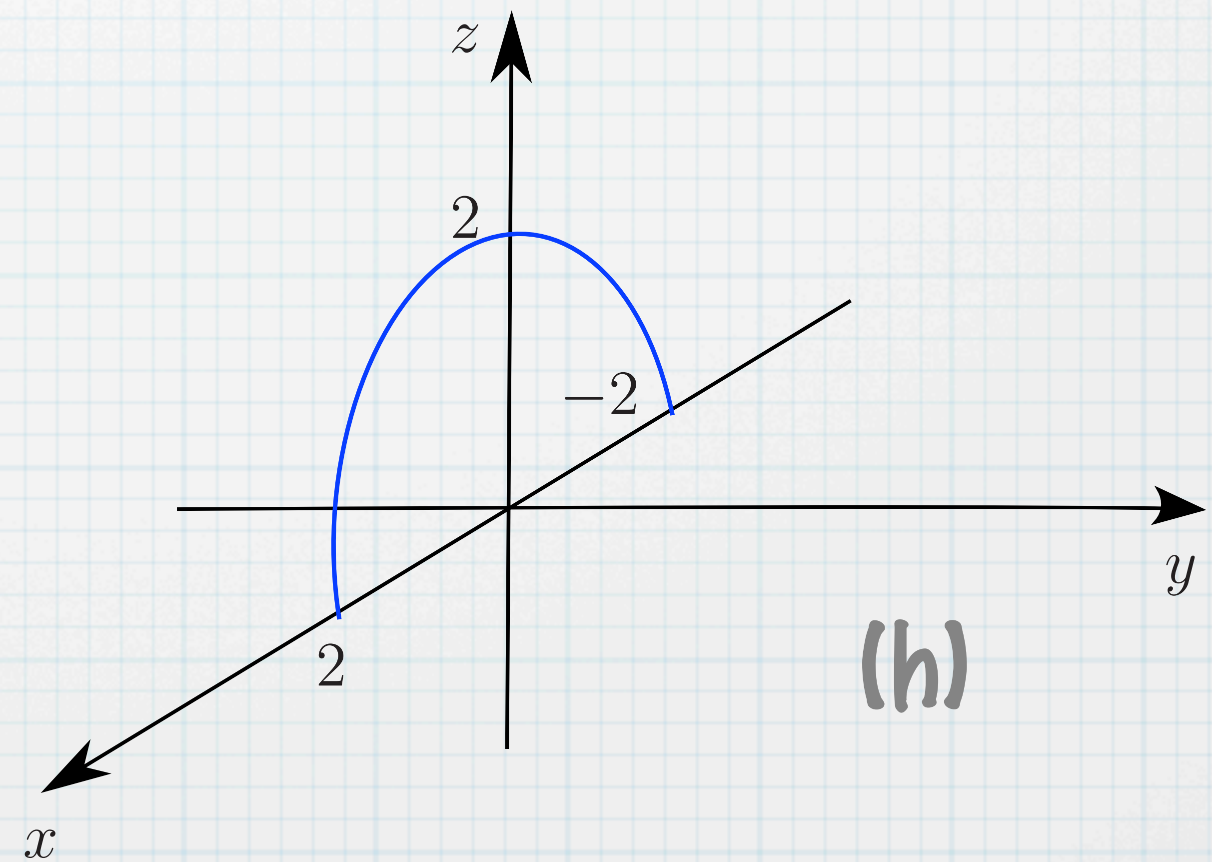
(e)



(f)



(g)



(h)

Ecuaciones vectoriales problema 1.

$1.- \quad \mathbf{r}(t) = t\mathbf{i} + (1-t)\mathbf{j},\ 0 \leq t \leq 1$

$2.- \quad \mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k},\ -1 \leq t \leq 1$

$3.- \quad \mathbf{r}(t) = (2\cos(t)\mathbf{i} + (2\mathrm{sen}(t))\mathbf{j},\ 0 \leq t \leq 2\pi$

$4.- \quad \mathbf{r}(t) = t\mathbf{i},\ -1 \leq t \leq 1$

$5.- \quad \mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k},\ 0 \leq t \leq 2$

$6.- \quad \mathbf{r}(t) = t\mathbf{j} + (2-2t)\mathbf{k},\ 0 \leq t \leq 1$

$7.- \quad \mathbf{r}(t) = (t^2-1)\mathbf{j} + (2t)\mathbf{k},\ -1 \leq t \leq 1$

$8.- \quad \mathbf{r}(t) = (2\cos(t)\mathbf{i} + (2\mathrm{sen}(t))\mathbf{k},\ 0 \leq t \leq \pi$

Respuesta:

1-c
2-e
3-g
4-a
5-d
6-b
7-f
8-h

Problema 2.

Evalúe $\int_C (x+y)\,ds$ **donde C es el segmento de linea de ecuación** $x = t,\ y = (1-t),\ z = 0,$ **desde (0,1,0) a (1,0,0).**

$$\mathbf{r}(\mathrm{t}) = \mathrm{t}\mathbf{i} + (1-\mathrm{t})\mathbf{j}\,,\ 0 \le \mathrm{t} \le 1$$

$$\Rightarrow\ \frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}} = \mathbf{i} - \mathbf{j}\ \Rightarrow\ \left|\frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}}\right| = \sqrt{2}\,\mathbf{j}$$

$$x = t,\ y = 1 - t\ \Rightarrow\ \mathrm{x} + \mathrm{y} = \mathrm{t} + (1 - \mathrm{t}) = 1$$

$$\Rightarrow \int_{\mathrm{C}} \mathrm{f}(\mathrm{x},\mathrm{y},\mathrm{z})\,\mathrm{ds} = \int_0^1 \mathrm{f}(\mathrm{t}, 1-\mathrm{t}, 0)\left|\frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}}\right|\mathrm{dt}$$

$$= \int_0^1 (1)\left(\sqrt{2}\right)\mathrm{dt} = \left[\sqrt{2}\,\mathrm{t}\right]_0^1 = \sqrt{2}$$

Problema 3.

Evalúe $\int_C \sqrt{x^2+y^2}\,ds$ a lo largo de la curva de ecuación

$$\mathbf{r}(t) = (4\cos(t))\mathbf{i} + (4\text{sen}(t))\mathbf{j} + 3t\mathbf{k},\ -2\pi \le t \le 2\pi$$

$$\mathbf{r}(\mathrm{t}) = (4\cos\mathrm{t})\mathbf{i} + (4\sin\mathrm{t})\mathbf{j} + 3\mathrm{t}\mathbf{k}\,, -2\pi \le \mathrm{t} \le 2\pi$$

$$\Rightarrow\ \frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}} = (-4\sin\mathrm{t})\mathbf{i} + (4\cos\mathrm{t})\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow\ \left|\frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}}\right| = \sqrt{16\sin^2\mathrm{t} + 16\cos^2\mathrm{t} + 9} = 5$$

$$\sqrt{\mathrm{x}^2+\mathrm{y}^2} = \sqrt{16\cos^2\mathrm{t} + 16\sin^2\mathrm{t}} = 4$$

$$\Rightarrow \int_C f(x, y, z)\, ds = \int_{-2\pi}^{2\pi} (4)(5)\, dt$$

$$= [20t]_{-2\pi}^{2\pi} = 80\pi$$

Problema 4.

Integre $f(x, yz) = x + \sqrt{y} - z^2$ desde el punto (0,0,0) al punto (1,1,1), por medio de las curvas:

$$C_1 : \ \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}, \ 0 \leq t \leq 1$$

$$C_2 : \ \mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}, \ 0 \leq t \leq 1$$

$$C_1: \ \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}\,,\ 0 \le t \le 1 \ \Rightarrow \ \frac{d\mathbf{r}}{dt} = \mathbf{i} + 2t\mathbf{j} \ \Rightarrow \ \left|\frac{d\mathbf{r}}{dt}\right| = \sqrt{1+4t^2}$$

$$x + \sqrt{y} - z^2 = t + \sqrt{t^2} - 0 = t + |t| = 2t$$

$$t \ge 0 \ \Rightarrow \int_{C_1} f(x,y,z)\,ds = \int_0^1 2t\sqrt{1+4t^2}\,dt$$

$$= \left[\frac{1}{6}\left(1+4t^2\right)^{3/2}\right]_0^1 = \frac{1}{6}(5)^{3/2} - \frac{1}{6} = \frac{1}{6}\left(5\sqrt{5}-1\right)$$

$$C_2\text{:}\ \mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k},\ 0 \le t \le 1 \ \Rightarrow\ \frac{d\mathbf{r}}{dt} = \mathbf{k} \ \Rightarrow\ \left|\frac{d\mathbf{r}}{dt}\right| = 1$$

$$x + \sqrt{y} - z^2 = 1 + \sqrt{1} - t^2 = 2 - t^2$$

$$\Rightarrow \int_{C_2} f(x, y, z)\, ds = \int_0^1 (2 - t^2)\,(1)\, dt = \left[2t - \tfrac{1}{3}\, t^3\right]_0^1 = 2 - \tfrac{1}{3} = \tfrac{5}{3}$$

$$\int_{C_1} f(x, y, z)\, ds + \int_{C_2} f(x, y, z)\, ds = \tfrac{5}{6}\sqrt{5} + \tfrac{3}{2}$$

Problema 5.

Encuentre la masa de un cable que yace a lo largo de la curva de ecuación

$$\mathbf{r}(t) = (t^2 - 1)\mathbf{j} + 2t\mathbf{k},\ \ 0 \leq t \leq 1$$

si la densidad esta dada por

$$\delta = \left(\frac{3}{2}\right) t$$

$$\mathbf{r}(t) = (t^2 - 1)\,\mathbf{j} + 2t\mathbf{k}\,,\, 0 \le t \le 1 \;\Rightarrow\; \frac{d\mathbf{r}}{dt} = 2t\mathbf{j} + 2\mathbf{k} \Rightarrow \left|\frac{d\mathbf{r}}{dt}\right| = 2\sqrt{t^2 + 1}$$

$$M = \int_C \delta(x, y, z)\, ds = \int_0^1 \delta(t) \left(2\sqrt{t^2 + 1}\right) dt$$

$$= \int_0^1 \left(\tfrac{3}{2}\, t\right) \left(2\sqrt{t^2 + 1}\right) dt = \left[(t^2 + 1)^{3/2}\right]_0^1 = 2^{3/2} - 1 = 2\sqrt{2} - 1$$